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Form Factors of Baryons in a Confining and Covariant Diquark-Quark Model[1]

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Abstract. We treat baryons as bound states of scalar or axialvector diquarks and a constituent quark which interact through quark exchange. This description results as an approximation to the relativistic Faddeev equation for three quarks which yields an effective Bethe-Salpeter equation. Octet and decuplet masses and fully four-dimensional wave functions have been computed for two cases: assuming an essentially pointlike diquark on the one hand, and a diquark with internal structure on the other hand. Whereas the differences in the mass spectrum are fairly small, the nucleon electromagnetic form factors are greatly improved assuming a diquark with structure. First calculations to the pion-nucleon form factor also suggest improvements.

I MOTIVATION

Two approaches to the rich structure of strong interaction phenomena have been the topic of this workshop. The first one, effective theories like Chiral Perturbation Theory, resorts to including only physical fields with a suitable expansion parameter. The second approach, the building of effective models, often tries to interpolate between QCD and observable degrees of freedom by taking loans from the latter in terms of the assumed relevant degrees of freedom, such as (constituent) quarks. Different types of these models describe various aspects of baryon physics. Among them are nonrelativistic quark models, various sorts of bag models and approaches describing baryons by means of collective variables like topological or non-topological solitons [1]. Most of these models are designed to work in the low-energy region and generally do

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not match the calculations within perturbative QCD. Considering the great experimental progress in the medium energy range with momentum transfers between 1 and 5 GeV^2, there is a high demand for models describing baryon physics in this region that connects the low and high energy regimes.

To describe this kind of physics, a fully covariant approach seems indispensable. Furthermore, the effects of quark confinement should be incorporated into a reliable description to avoid unphysical break-ups of baryons into their constituents. This is in sharp contrast to low-energy or static observables: baryon masses and magnetic moments, *e.g.*, can be understood in terms of a dynamically generated constituent quark mass through chiral symmetry breaking. Confinement plays seemingly an unimportant role.

The Nambu-Jona-Lasinio model in its various guises shows this feature of a dynamically generated quark mass and has thus been utilized to describe mesonic properties quite successfully [2]. The description of baryons within this model allows for two possibilities: They may appear as non-topological solitons [3,4] or as bound states of quark and diquark [5]. In ladder approximation, diquarks appear as poles in quark-quark scattering and therefore as physical particles. They are confined when going beyond ladder approximation [6]. A study which incorporates both, solitons and diquark-quark bound states [7], shows that the mesonic cloud and the quark-diquark interaction contribute about equally to the binding energy of the baryon.

On the other hand, the relativistic three-body problem can be simplified when discarding three-body irreducible interactions. The resulting Faddeev-type problem can be reduced further by assuming separable two-quark correlations which are usually called diquarks [8,9]. The Faddeev equations then collapse to a Bethe-Salpeter equation whose solutions describe the baryons. Quark and diquark hereby interact through quark exchange which restores full antisymmetry between the three quarks[2]. It is interesting to note that within the NJL model the two-quark correlations (or 4-point quark Green function) are separable in first order to yield a sum over poles of diquarks with different quantum numbers. In analogy to the meson spectrum[3], scalar and axialvector diquarks are assumed to be the lowest-lying and thus the most important particles. This line of approach has been taken in [9].

[2] Due to antisymmetry in the color indices and the related symmetrization of all other quantum numbers the Pauli principle leads to an attractive interaction in contrast to "Pauli repulsion" known in conventional few-fermion systems.
[3] Scalar diquarks correspond to pseudoscalar mesons and axialvector diquarks to vector mesons due to the intrinsically different parity of a fermion-antifermion pair compaired to a fermion pair.

II THE MODEL

In the subsequent sections, we will follow this approach and derive an effective baryon Bethe-Salpeter equation with quark and diquark as constituents. However, to mimic confinement, we will avoid the diquark poles which would correspond to unphysical thresholds. To this end, consider the 4-point quark Green function in coordinate space,

$$G_{\alpha\beta\gamma\delta}(x_1, x_2, x_3, x_4) = \langle T(q_\gamma(x_3) q_\alpha(x_1) \bar{q}_\beta(x_2) \bar{q}_\delta(x_4)) \rangle \ , \tag{1}$$

where α, β, γ, and δ denote the Dirac indices of the quarks. Assuming this 4-point function to be separable, we will parameterize scalar and axialvector diquark correlations as:

$$\begin{aligned} G^{\mathrm{sep}}_{\alpha\gamma,\beta\delta}(p, q, P) &:= e^{-iPY} \int d^4X\, d^4y\, d^4z\ e^{iqz} e^{-ipy} e^{iPX} G^{\mathrm{sep}}_{\alpha\beta\gamma\delta}(x_1, x_2, x_3, x_4) \\ &= \chi_{\gamma\alpha}(p)\, D(P)\, \bar{\chi}_{\beta\delta}(q) \ + \ \chi^\mu_{\gamma\alpha}(p)\, D^{\mu\nu}(P) \bar{\chi}^\nu_{\beta\delta}(q) \ , \end{aligned} \tag{2}$$

P is the total momentum of the incoming and the outgoing quark-quark pair, p and q are the relative momenta between the quarks in these channels as y and z are the relative coordinates.

$\chi_{\alpha\beta}(p)$ and $\chi^\mu_{\alpha\beta}(p)$ are vertex functions of quarks with a scalar and an axialvector diquark, respectively. They belong to a $\bar{3}$-representation in color space and are flavor antisymmetric (scalar diquark) or flavor symmetric (axialvector diquark). For their Dirac structure we will retain the dominant contribution only, and a scalar function $P(p)$ which depends only on the relative momentum p between the quarks parameterizes the extension of the vertex in momentum space[4]:

$$\chi_{\alpha\beta}(p) = g_s (\gamma^5 C)_{\alpha\beta}\ P(p) \ , \tag{3}$$

$$\chi^\mu_{\alpha\beta}(p) = g_a (\gamma^\mu C)_{\alpha\beta}\ P(p). \tag{4}$$

C denotes hereby the charge conjugation matrix and g_a and g_s are normalization constants at this stage. The choice

$$P(p) = 1 \tag{5}$$

corresponds to a point-like diquark whereas extended diquarks can be modeled as

$$P(p) = \left(\frac{\gamma^2}{\gamma^2 + p^2} \right)^n . \tag{6}$$

This specific form with n=2 or n=4 proved to be quite successful in describing electromagnetic properties of the nucleon when using scalar diquarks only [10].

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[4] The Pauli principle requires then the relative momentum to be defined $p = \frac{1}{2}(p_\alpha - p_\beta)$, where p_α and p_β are the quark momenta [10].

To parameterize confinement, the propagators of scalar and axialvector diquark, appearing in eq. (2) as $D(P)$ and $D^{\mu\nu}(P)$, ought to be modified. Our chosen form,

$$D(p) = -\frac{1}{p^2+m_{sc}^2}\left(1-e^{-\left(1+\frac{p^2}{m_{sc}^2}\right)}\right), \quad (7)$$

$$D^{\mu\nu}(p) = -\frac{\delta^{\mu\nu}}{p^2+m_{ax}^2}\left(1-e^{-\left(1+\frac{p^2}{m_{ax}^2}\right)}\right), \quad (8)$$

removes the free particle poles at the cost of an essential singularity for timelike infinitely large momenta. The constituent quark propagator is modified likewise:

$$S(p) = \frac{i\not{p}-m_q}{p^2+m_q^2}\left(1-e^{-\left(1+\frac{p^2}{m_q^2}\right)}\right). \quad (9)$$

With these ingredients, the Faddeev equations for the three quark system can be simplified enormously. To do this, one enters the Faddeev equations with an ansatz for the truncated, irreducible 3-quark correlation function (the 6-point quark Green function), which, *e.g.*, exhibits a pole from a spin-1/2 baryon:

$$G^{trunc}_{\alpha\beta\gamma,\delta\epsilon\zeta} \sim \frac{\Gamma_{\alpha\beta\gamma}(P;p,p_d,p_1)\,\bar{\Gamma}_{\delta\epsilon\zeta}(P;q,q_d,q_1)}{P^2+M^2}, \quad (10)$$

$$\Gamma_{\alpha\beta\gamma} = \chi_{\beta\gamma}(p_1)D(p_d)(\Phi^5(P,p)u)_\alpha + \chi^\mu_{\beta\gamma}(p_1)D^{\mu\nu}(p_d)(\Phi^\nu(P,p)u)_\alpha \quad (11)$$

The flavor and color indices which have to be found after projection onto the baryon quantum numbers have been omitted here. The object of interest is now the nucleon vertex function $\Phi u = \begin{pmatrix}\Phi \\ \Phi^\mu\end{pmatrix} u$ (with u being a positive-energy Dirac spinor) which represents an effective spinor characterizing the scalar and the axialvector diquark correlations within the nucleon.

For this effective spinor, a coupled set of Bethe-Salpeter equations can be derived. Its complete derivation can be found in [9]. For spin-1/2 baryons in the flavor-symmetric case, the equation takes the form:

$$\begin{pmatrix}\Psi^5 \\ \Psi^{\mu'}\end{pmatrix}(p,P) := S(p_q)\begin{pmatrix}D & 0 \\ 0 & D^{\mu'\mu}\end{pmatrix}(p_d)\begin{pmatrix}\Phi^5 \\ \Phi^\mu\end{pmatrix}(p,P) \quad (12)$$

$$\begin{pmatrix}\Phi^5 \\ \Phi^\mu\end{pmatrix}(p,P) = \int\frac{d^4p'}{(2\pi)^4}\frac{1}{2}\begin{pmatrix}-\chi S^T(q)\bar{\chi} & \sqrt{3}\chi^{\mu'}S^T(q)\bar{\chi} \\ \sqrt{3}\chi S^T(q)\bar{\chi}^\mu & \chi^{\mu'}S^T(q)\bar{\chi}^\mu\end{pmatrix}\begin{pmatrix}\Psi^5 \\ \Psi^{\mu'}\end{pmatrix}(p',P).$$

It is pictorially represented in Fig. 1. The attraction that leads to a bound state is the quark exchange between the two constituents. Note that we banned all unknown and possibly very complicated gluonic interactions between the quarks into the parameterization of the two-quark correlations. The quark exchange is a consequence of the structure of the Faddeev equations.

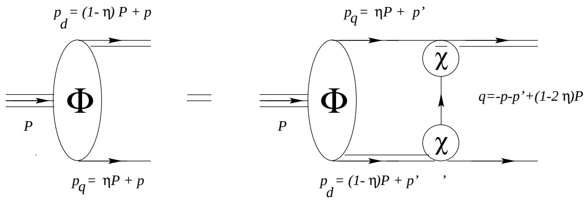


FIGURE 1. The baryon Bethe-Salpeter equation. The momentum partitioning parameter η distributes the relative momentum p' over quark and diquark.

The quark-diquark vertex from eqs. (3, 4) enters as the quark-diquark interaction vertex. This equation can be solved without any further approximation, especially without any non-relativistic reduction. First one decomposes the baryon vertex Φ (where each component is a 4×4-matrix) in Dirac space and projects onto positive parity and energy states. This procedure is described in detail in [11]. Choosing the rest frame of the nucleon, all independent components are regrouped as eigenstates of orbital angular momentum. As a final result, eight independent amplitudes, *i.e.* scalar functions which multiply the components, describe the spin-1/2 baryon as can be seen from Table 1. As the amplitudes still depend on two momenta (the relative momentum p and the total momentum P), an expansion in terms of Chebyshev polynomials for the variable $p\cdot P/(|p||P|)$ is performed. Thus the four-dimensional equation (12) can be reduced to a number of coupled one-dimensional integral equations [11,10] which we solved iteratively.

This procedure can be applied to spin-3/2 baryons as well [11]. Again eight independent amplitudes are found after spin and energy projection. Here, as a difference to spin-1/2 baryons, only one s partial wave exists which is found to dominate the expansion.

TABLE 1. Components of the octet baryon vertex function with their respective spin and orbital angular momentum. $(\gamma_5 C)$ corresponds to scalar and $(\gamma^\mu C)$, $\mu = 1\ldots4$, to axialvector diquark correlations. Note that the partial waves in the first row possess a non-relativistic limit.

"non-relativistic" partial waves	$\begin{pmatrix}\chi\\0\end{pmatrix}(\gamma_5 C)$	$\hat{P}^4\begin{pmatrix}0\\\chi\end{pmatrix}(\gamma^4 C)$	$\begin{pmatrix}i\sigma^i\chi\\0\end{pmatrix}(\gamma^i C)$	$\begin{pmatrix}i\left(\hat{p}^i(\vec{\sigma}\hat{\vec{p}})-\frac{\sigma^i}{3}\right)\chi\\0\end{pmatrix}(\gamma^i C)$
spin	1/2	1/2	1/2	3/2
orbital angular momentum	s	s	s	d
"relativistic" partial waves	$\begin{pmatrix}0\\\vec{\sigma}\vec{p}\chi\end{pmatrix}(\gamma_5 C)$	$\hat{P}^4\begin{pmatrix}(\vec{\sigma}\vec{p})\chi\\0\end{pmatrix}(\gamma^4 C)$	$\begin{pmatrix}0\\i\sigma^i(\vec{\sigma}\vec{p})\chi\end{pmatrix}(\gamma^i C)$	$\begin{pmatrix}0\\i\left(p^i-\frac{\sigma^i(\vec{\sigma}\vec{p})}{3}\right)\chi\end{pmatrix}(\gamma^i C)$
spin	1/2	1/2	1/2	3/2
orbital angular momentum	p	p	p	p

TABLE 2. Octet and decuplet masses.

	exp.	pointlike	extended diquark
		$P(p) = 1$	$P(p) = \left(\dfrac{\gamma^2}{\gamma^2 + p^2}\right)^4$ $\gamma = 0.5$ GeV
m_u (GeV)		0.5	0.56
m_s (GeV)		0.63	0.68
ξ		0.73	0.6
M_Λ (GeV)	1.116	1.133	1.098
M_Σ (GeV)	1.193	1.140	1.129
M_Ξ (GeV)	1.315	1.319	1.279
M_{Σ^*} (GeV)	1.384	1.380	1.396
M_{Ξ^*} (GeV)	1.530	1.516	1.572
M_Ω (GeV)	1.672	1.665	1.766

III RESULTS FOR OBSERVABLES

A Octet and Decuplet Masses

In our approach the strange quark constituent mass m_s is the only source of flavour symmetry breaking. Isospin is assumed to be conserved. The equations describing octet and decuplet baryons have been derived under the premises of flavour and spin conservation, *i.e.* only vertex function components with same spin and flavour content couple. Again the full set of equations can be found in [11]. The results for the cases of a pointlike diquark and an extended diquark are shown in Tab. 2. We chose scalar and axialvector diquark masses[5] to be equal and proportional to the sum of the two quark masses constituting the diquark. The proportionality constant is called ξ. The nucleon and the delta mass served as input to determine the normalization constants g_s and g_a appearing in eqs. (3,4). From the viewpoint of the effective quark-diquark theory, g_s and g_a reflect the coupling strengths in the two diquark channels.

As can be seen from the numbers, the mass splitting between octet and decuplet can be explained as a result of the relativistic dynamics only. In the case of extended diquarks, the splitting is even overestimated.

[5] The use of confining propagators renders the masses to be mere parameters which set the scale in the propagators, eqs. (7-9). They are of course unobservable.

B Electromagnetic Form Factors

Calculation of observables within the Bethe-Salpeter framework proceeds along Mandelstam's formalism [12]. The two necessary ingredients are normalized nucleon-quark-diquark vertex functions and, in case of the electromagnetic form factors, the current operator. The vertex functions can be calculated as outlined in the previous section and their normalization is determined by the canonical normalization to the correct (fermionic) bound state residue, see, *e.g.*, [13]. To this end, we define an object $G(p,p',P)$ involving the quark and diquark propagators and the exchange kernel appearing in the Bethe-Salpeter equation (12),

$$\begin{aligned} G(p,p',P) &= (2\pi)^4\delta(p-p')S^{-1}(p_q)\begin{pmatrix} D^{-1} & 0 \\ 0 & (D^{\mu'\mu})^{-1}\end{pmatrix}(p_d) + \\ &\quad \frac{1}{2}\begin{pmatrix} \chi S^T(q)\bar{\chi} & -\sqrt{3}\chi^{\mu'}S^T(q)\bar{\chi} \\ -\sqrt{3}\chi S^T(q)\bar{\chi}^{\mu} & -\chi^{\mu'}S^T(q)\bar{\chi}^{\mu}\end{pmatrix}. \end{aligned} \tag{13}$$

With Λ^+ being the positive-energy projector, the normalization condition is:

$$-\int\frac{d^4p}{(2\pi)^4}\int\frac{d^4p'}{(2\pi)^4}\bar{\Psi}(p',P_n)\left[P^\mu\frac{\partial}{\partial P^\mu}G(p,p',P)\right]_{P=P_n}\Psi(p,P_n)\stackrel{!}{=}M\Lambda^+. \tag{14}$$

The current operator consists of the couplings of the photon to quark and diquark (impulse approximation) and to the exchange kernel G. For extended diquarks, it has been shown in [10] that the latter contribution encompasses two parts to make the total baryon current transversal and to reproduce the correct charge. These two parts are the interaction of the photon with the exchanged quark and its coupling to the diquark-quark vertex χ or χ^μ that can be described by a seagull-like photon-quark-diquark vertex. In the case of pointlike diquarks, this seagull contribution vanishes.

To summarize, one has to calculate the diagrams given in Fig. 2. To ensure gauge invariance, the quark-photon and the diquark-photon vertices are of the Ball-Chiu type [14,15]. The seagull vertex is given by

$$\Gamma^\mu = e_\alpha\frac{4p^\mu - Q^\mu}{4pQ-Q^2}\left[\chi\left(p-\frac{Q}{2}\right)-\chi(p)\right]-\begin{pmatrix}\alpha\to\beta\\ Q\to -Q\end{pmatrix}. \tag{15}$$

An analogous relation is valid for the seagull involving the axialvector diquark vertex χ^ν. As before, p is the relative momentum between the two quarks, and e_α, e_β denote their respective charges.

We computed the Sachs form factors G_E and G_M for proton and neutron using the parameters given in Tab. 2. The results for the electric form factor are shown in Fig. 3. Clearly, the proton curve falls too weakly for a pointlike diquark which signals that the nucleon-quark-diquark vertex has too small a size in coordinate space. This is remedied by the introduction of the diquark structure. However, the neutron electric form factor seems to be quenched

FIGURE 2. Diagrams that built up the baryon matrix elements of the electromagnetic current. The first row shows the diagrams of the impulse approximation, the second row the contributions of the exchange kernel.

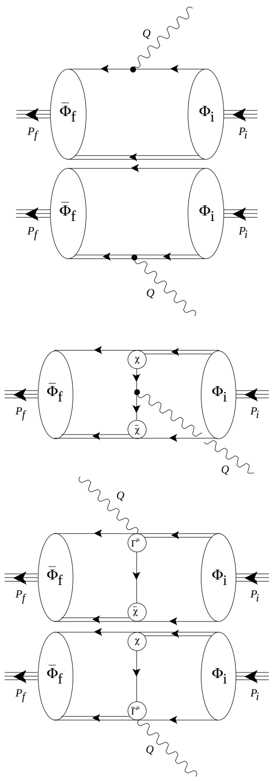
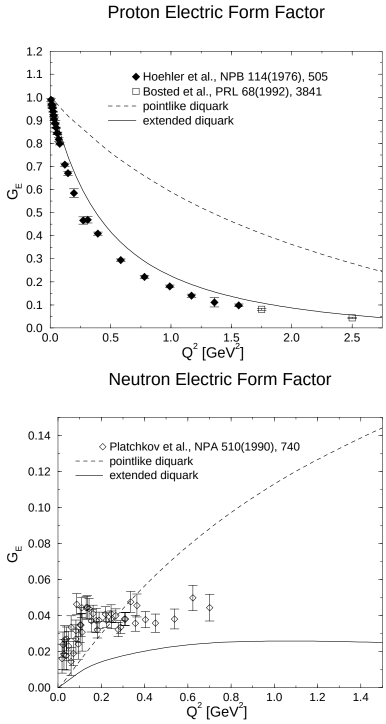


FIGURE 3. The electric form factor of proton and neutron.



too strongly as compared to the data[6]. Now this problem is probably due to overestimated axialvector diquark correlations within the nucleon. Retaining extended scalar diquarks only yields a very satisfactory description of the neutron G_E [10].

The nucleon magnetic moments have also improved with the introduction of the extended diquarks, see Fig. 4. Nevertheless, their absolute values are still about 13% too small in comparison with experiment although the ratio μ_p/μ_n is reproduced nicely. In our formalism, the diquarks have no anomalous magnetic moments since we do not properly resolve the diquark in the second impulse approximation diagram of Fig. 2. Performing Mandelstam's formalism for the diquark itself, *i.e.* coupling the photon to each of the quarks and letting them recombine to the diquark, would therefore certainly improve on the magnetic moments. In Fig. 4 we have also plotted separately the contributions of the impulse approximation and of the coupling to the exchange kernel. As the second contribution makes up more than 30 per cent of the total magnetic moment, the less involved impulse approximation is merely a rough guide to the behaviour of the magnetic form factor.

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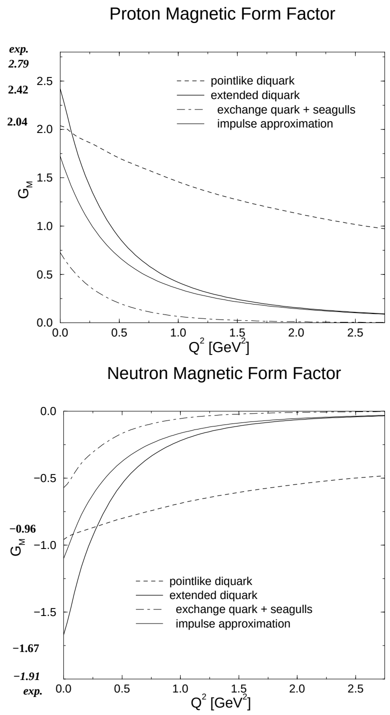
[6]) As has been pointed out in [16], these data should not be over-interpreted as systematic errors have been involved in extracting them from raw data. Nevertheless they give a feeling for the qualitative behaviour of the form factor.

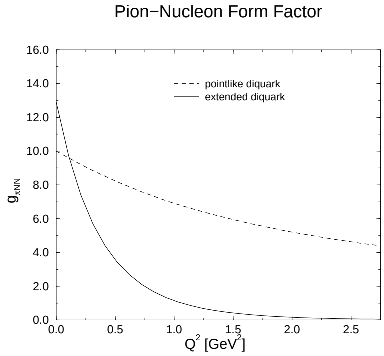
FIGURE 4. The magnetic form factor of proton and neutron.



C Strong Form Factors

Among various strong processes that are candidates for closer scrutiny within our model, we have chosen first the pion-nucleon form factor $g_{\pi NN}(Q^2)$. Hereby we couple the pion to the quark only with its dominant Dirac amplitude $\sim \gamma_5$. This is certainly a good approximation as more detailed, microscopic calculations have shown [17]. The on-shell pion-quark vertex is dictated by PCAC and for the off-shell extrapolation we used a form proposed by ref. [18] and which has been applied in [15]. In our model, the diquark contributes nothing to $g_{\pi NN}$. This is a simple consequence of the Dirac algebra if one tries to couple the pion to each of the two quarks within the diquark. The results for the impulse approximation diagram only is shown in Fig. 5. Again, the fall-off in the case of the pointlike diquark is much slower than a monopole and appears to be unphysical. In contrast to this, $g_{\pi NN}$ for the extended diquark falls slightly stronger than a monopole with a width parameter of around 360 MeV. In the light of the results for the magnetic moments, the value of $g_{\pi NN}$ at $Q = 0$ may still be subject to sizeable corrections coming from the coupling to the exchange quark.

FIGURE 5. The strong form factor $g_{\pi NN}$.



IV CONCLUSION

We have suggested a field theoretic model of baryons that makes use of diquarks which are a parameterization of the quark-quark correlations within baryons. Thereby we could retain full covariance. We parameterized confinement by a suitable modification of quark and diquark propagators to avoid unphysical thresholds.

Masses and four-dimensional vertex functions have been calculated for the baryon octet and decuplet. These vertex functions are the main ingredient for the calculation of observables such as the nucleon electromagnetic form factors. Whereas the mass spectrum is quite unsensitive to the extension of the diquarks, the form factors provide an effective mean to fix it. In these calculations gauge invariance was strictly maintained. However, the nucleon magnetic moments are still about 15 per cent to small. This we attribute to our incomplete handling of the electromagnetic structure of the diquark.

The computation of the pion-nucleon form factor is a necessary intermediate step to calculate production processes. As the pseudoscalar mesons do not couple to the diquarks, these processes are particularly transparent within the framework of our model. Additionally, a Λ hyperon in the final state renders the flavor algebra simple, therefore we have chosen associated strangeness production ($pp \to pK\Lambda$) and kaon photoproduction ($\gamma p \to K\Lambda$) as further testing ground for our model [19].

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REFERENCES

1. *see e.g.* Bhaduri, R., *Models of the Nucleon*, New York: Addison-Wesley, 1988.

2. *see e.g.* Ebert, D., *et al.*, *Prog. Part. Nucl. Phys.* **33**, 1 (1994).
3. Alkofer, R., Reinhardt, H., and Weigel, H., *Phys. Rep.* **265**, 139 (1996).
4. Christov, C. V., *et al.*, *Prog. Part. Nucl. Phys.* **37**, 1 (1996).
5. Reinhardt, H., *Phys. Lett. B* **244**, 316 (1990), *see also* Alkofer, R., and Reinhardt, H., *Chiral Quark Dynamics*, Heidelberg: Springer, 1995.
6. Hellstern, G., Alkofer, R., and Reinhardt, H., *Nucl. Phys. A* **625**, 697 (1997).
7. Zückert, U., *et al.*, *Phys. Rev. C* **55** 2030, (1997).
8. Burden, C. J., Cahill, R. T., and Praschifka, J., *Aust. J. Phys.* **42**, 147 (1989).
9. Ishii, N., Bentz, W., and Yazaki, K., *Nucl. Phys. A* **51**, 617 (1995).
10. Oettel, M., Pichowsky, M. A., and von Smekal, L., *nucl-th/9909082.*
11. Oettel, M., *et al.*, *Phys. Rev. C* **58** 2459, (1998).
12. Mandelstam, S., *Proc. Roy. Soc. A* **233**, 248 (1955).
13. Itzykson, C., and Zuber, B., *Quantum Field Theory*, New York: McGraw-Hill, 1985.
14. Ball, J. S., and Chiu, T.-W., *Phys. Rev. D* **22**, 2542 (1980).
15. Hellstern, G., *et al.*, *Nucl. Phys. A* **627**, 679 (1997).
16. Passchier, I., *et al.*, *Phys. Rev. Lett.* **82**, 4988 (1999).
17. *see e.g.* Tandy, P. C., *Prog. Part. Nucl. Phys.* **39**, 117 (1997).
18. Delbourgo, R., and Scadron, M. D., *J. Phys. G* **5**, 1621 (1979).
19. Alkofer, R., *Talk given at MENU99, to appear in the Proceedings.*